configurations are summarized in Tables 1 and 2 for the 10 in. and 23 in. diameter models, respectively. In both tables it is seen that the effect of c.g. location is that more forward locations increase the static stability. This fact is indicated (on the assumption that the disturbances in the wind stream are the same for all models) by the perturbations of the models from the zero degree trim angle reducing as the c.g. is moved forward. In Tables 1 and 2 the effects of ring diameter and porosity are actually inseparable; however, in Table 2, it is seen that for the lowest porosity (largest D/d) a finite limit cycle angle occurred and for the highest porosity (smallest D/d) a finite trim angle occurred, whereas in Table 1, with D/d of 1.105 [which is smaller than the smallest in Table 2 (1.113)] a greater porosity (47.7% compared to 22.1) resulted in changing the trim angle from 5° to 0°. Thus it appears that some minimum value of D/d produces limit cycling which is akin to the behavior of a plain sphere, and some minimum value of porosity produces a finite trim angle. It appears in Table 2 that the effect of increased porosity for the last entry tended to reduce the finite trim angle. The reason for no perturbations for this configuration is not known.

The perturbations indicated in the table do not seem to represent a systematic phenomenon. These perturbations were random in occurrence and are believed to be caused by random turbulence in the wind stream. The drag coefficients from Tables 1 and 2 are compared with the static data in Fig. 2.

Pitch damping coefficients deduced from the Spin Tunnel results for several configurations are shown in Fig. 4. It is interesting that, for these models, the damping coefficient increases strongly as angle of attack approaches zero, whereas, for most high-drag conical configurations, the damping decreases in this range of angle of attack and actually changes sign, resulting in limit cycle behavior. The canted holes in the rings served to effectively control the model roll rate. The roll helix angles measured were actually somewhat greater than the hole cant angles.

Low-Altitude Roll Behavior of Entry Vehicles with Mass Asymmetries

W. J. Bootle*
AVCO Systems Division, Wilmington, Mass.

It has been shown that a principle axis misalignment and an orthogonal c.g. offset will promote steady resonance in a slender entry vehicle at first intersection of the spin and pitch frequencies whenever the product of the two asymmetries exceeds a critical level K_{cr}.[1] The principle axis tilt is equivalent to a trim angle that is amplified at resonance crossover to produce a roll acceleration $\dot{p}$ that exceeds $\dot{\omega}_c$ and thereby ensures a lock-in.

If the combination is below the critical threshold, the spin rate merely shows a minor fluctuation and then levels off at the initial value p_o, while the angle of attack exhibits the characteristic high-altitude transient resonance divergence. The excursion peak lags the crossover altitude by an interval ΔH that according to a linear analysis by Kanno[2] depends upon the magnitude of the parameter λ where $\lambda = p_o(1-I_x/2I_y)/\beta V_E \sin\gamma_E$. This interval expressed in the non-dimensional form $\sigma = -\beta\Delta H$ is shown as a function of λ in Fig. 1, where β is the density scale height (22000 ft^{-1}). Six-degrees-of freedom simulations show close agreement with the linear analysis, and also show that to first order the lag is insensitive to C_{m_q}.[3] Typically, ΔH ranges from 5000 to 20000 ft depending on the spin rate and trajectory.

Received May 5, 1975. This study was performed in part under Air Force Contract AF04(694)-913 (RVTO).

Index categories: Missile Systems; Entry Vehicle Dynamics and Control.

*Senior Consulting Scientist, Vehicle Design Group, Applied Technology Directorate. Member AIAA.

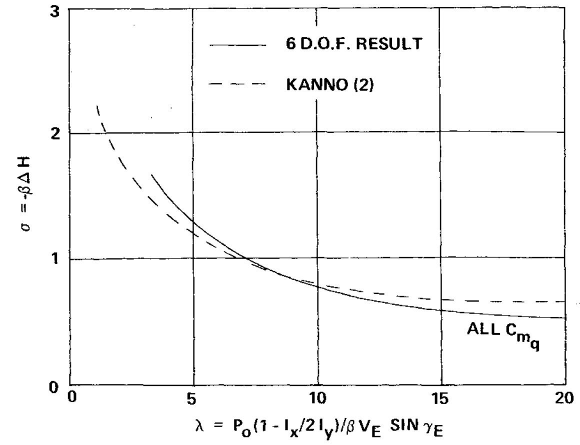

Fig. 1 Nondimensional transient resonance lag at first crossover vs λ.

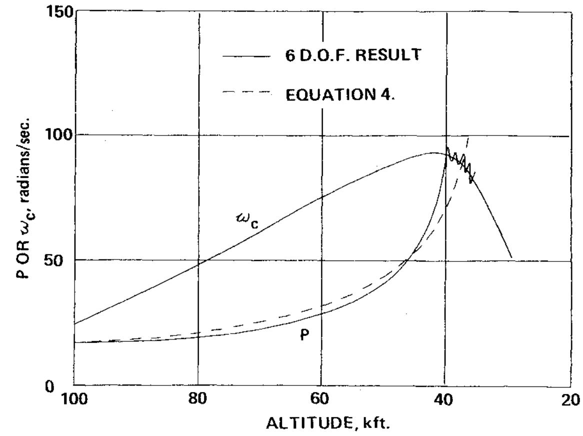

Fig. 2 Comparison of analytical solution for spin history with six-degrees-of-freedom computer results,

Although a lock-in may not occur, the subsequent spin history is important because this asymmetry combination may still cause a progressive spin up and a second intersection followed by the Pettus[4] type of low-altitude steady resonance to which the vehicle is much more prone because of the larger trim amplification and roll torques. A simple analysis for the spin history following initial crossover will now be presented.

Using the same nomenclature as before,[1] the spin acceleration is written thus:

$$\dot{p} = R\, C_{N_\alpha} q\, S(\alpha_T \Delta y - \beta_T \Delta z)/I_x \tag{1}$$

where R is the amplification ratio and α_T and β_T are the equivalent static trims generated by the principle axis tilt, viz.

$$\alpha_T = -(p/\omega_c)^2 I_{xz}/I_y - I_x \tag{2a}$$

$$\beta_T = -(p/\omega_c)^2 I_{xy}/I_y - I_x \tag{2b}$$

If the damping terms are ignored then $R = 1/(1-p^2/\omega_c^2)$ (not valid at $p=\omega_c$); over the region of interest $p^2/\omega_c^2 \ll 1$ so we may set $R=1$ without significant loss of accuracy. Then, substituting for $\omega_c^2 = c_{m\alpha} q\, SD/I_y - I_x$ and noting that $C_{m\alpha} = -C_{N\alpha}\bar{X}/D$ where $\bar{X}$ is the dimensional static margin, Eq. (1) reduces to

$$\dot{p}/p^2 = (-I_{xz}\Delta y + I_{xy}\Delta z)/I_x\bar{X} \tag{3}$$

This equation is then integrated with respect to time from the peak excursion altitude (not the crossover altitude) and the following result is obtained

$$1/p = 1/p_o + (I_{xz}\Delta y - I_{xy}\Delta z)\, t / I_x \bar{X} \tag{4}$$

Depending on the sign of the asymmetries it is seen that a spin or spin down will occur, but that a roll through zero ($p = 0$) is not possible within a finite time span; inertial asymmetries alone can only induce a roll near zero, and the associated dispersion is negligible since the equivalent static trims tend to zero [see Eq. (2)]. Equation 4 gives good agreement with the spin histories obtained on the six-degrees-of-freedom computer program for vehicles with this type of compound asymmetry. A typical comparison for a spin up combination that produced low altitude lock-in is shown in Fig. 2. The equation slightly underestimates the slope of the spin curve just prior to the second intersection because near resonant amplification of the principle axis trim is excluded when setting R at unity. However, the error is small because of the response lag.

Thus, if the pitch frequency history is known, the time to impact from resonance crossover, and the corresponding time from the transient peak may be estimated using Fig. 1. Substitution for t into Eq. (4) then provides the spin rate p at impact, and if this exceeds the pitch frequency a low-altitude crossover is indicated.

References

[1] Bootle, W. J., "Spin Variations in Slender Entry Vehicles During Rolling Trim," *AIAA Journal*, Vol. 9, April 1971, pp. 729-731.

[2] Kanno, J. S., "Spin-Induced Forced Resonant Behavior of a Ballistic Missile Reentering the Atmosphere," LMSD-288139, Vol. III, Jan. 1960, Lockheed Aircraft Corp., Missiles & Space Div., Sunnyvale, Calif.

[3] Bootle, W. J. and Kuczkowski, T., "Effects of C_{mq} on Transient Resonance Trim Amplification," TR-S240-67-WB-TK-70, June 1967, Avco Systems Div., Wilmington, Mass.

[4] Pettus, J. J., "Persistent Re-Entry Vehicle Roll Resonance," AIAA Paper 66-49, New York, 1966.